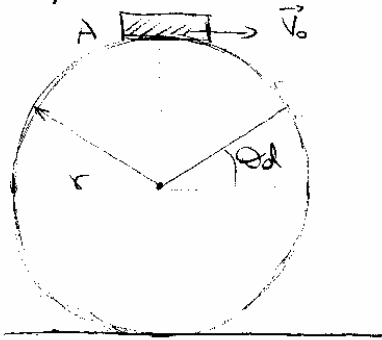


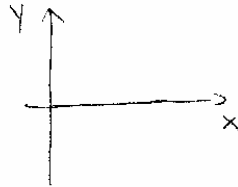
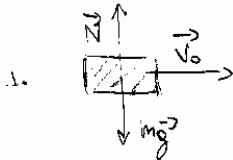
17/03/06 esercizio. (3 ore)



SFERA LISCIA

1. determinare il valore v^* del modulo di $|\vec{v}_0|$ al di sopra del quale il blocchetto si stacca subito nel punto A

2. se $|\vec{v}_0| < v^*$ determinare l'angolo θ_d a cui avviene il distacco



in A

$$\hat{x}: 0 = m a_x$$

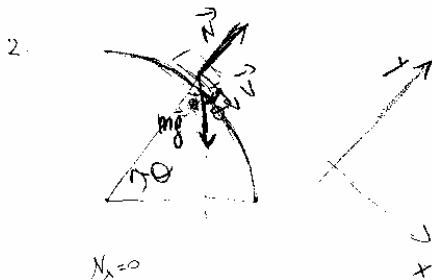
$$\hat{y}: N - mg = m a_y$$

$$a_y = a_{centrifuga} = -\frac{v^2}{r}$$

nella condizione limite $N=0$

perindi

$$0 - mg = -m \frac{v^{*2}}{r} \Rightarrow v^* = \sqrt{gr}$$



$$N_x = 0$$

$$\hat{x}: mg \cos \theta = m a_x$$

$$\hat{y}: N - mg \sin \theta = m a_y$$

$$a_y = -\frac{(v(\theta))^2}{r}$$

quando $\theta = \theta_d$ $N=0$

$$0 - mg \sin(\theta_d) = -m \frac{(v(\theta_d))^2}{r}$$

$v(\theta)$ dipende sia da θ
che da v_0 .

$$E_0 = \frac{1}{2} m v_0^2 + mg 2r = E_{\theta_d} = \frac{1}{2} m (v(\theta_d))^2 + mg(r + r \sin \theta_d)$$

o con il teorema delle forze vive:

$$W_{A \rightarrow B} = \Delta E_c^{A \rightarrow B}$$

$$\Delta E_c^{A \rightarrow B} = \frac{1}{2} m (v(\theta_d))^2 - \frac{1}{2} m v_0^2$$

$$W_{A \rightarrow B} = \int_A^B \vec{F} \cdot d\vec{s} = \int_A^B (mg\vec{j} + \vec{N}) \cdot d\vec{s}$$

essendo $\vec{N} \perp d\vec{s}$
 $\Rightarrow \vec{N}$ non compie lavoro

$$W_{AB} = \int_A^B mg\vec{j} \cdot d\vec{s} = \int_{\frac{\pi}{2}}^{\theta_d} -mg \cos \theta r d\theta = mgr (1 - \sin \theta_d)$$

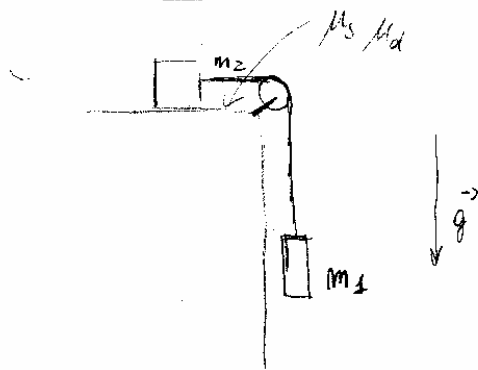
diminuendo l'angolo
aumenta ds

$$\begin{cases} \frac{1}{2} m v(\theta_d)^2 = \frac{1}{2} m v_0^2 + mgr (1 - \sin \theta_d) \\ (v(\theta_d))^2 = rg \sin \theta_d \end{cases}$$

$$\frac{1}{2} m rg \sin \theta_d = \frac{1}{2} m v_0^2 + mgr (1 - \sin \theta_d)$$

$$\sin \theta_d = \frac{v_0^2}{3rg} + \frac{2}{3}$$

Esercizio 2

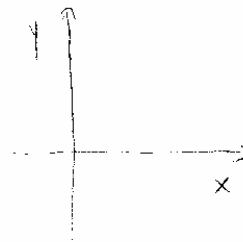
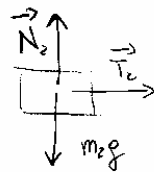


1) $\mu_s = \mu_d = 0$ descrivere moto di m_1 e m_2

2) μ_s, μ_d condizione su μ_s (valore minimo di μ_s) perché m_2 comincia a scivolare (μ_s^*)

3) $\mu_s < \mu_s^*$ descrivere il moto di m_2 e m_1

1. moto = saper scrivere l'equaz. del moto



①

$$\hat{x}: 0 = m_2 a_{2x}$$

$$\hat{y}: T_1 - m_1 g = m_1 a_{1y}$$

②

$$\hat{x}: T_2 = m_2 a_{2x}$$

$$\hat{y}: N_2 - m_2 g = m_2 a_{2y} = 0 \quad \Rightarrow \quad N_2 = m_2 g$$

perché $a_{2y} = 0$

$$|\vec{T}_1| = |\vec{T}_2|$$

essendo il filo inest.

i due blocchetti avranno la stessa velocità in modulo, quindi avranno la stessa ~~velocità~~ in modulo acceler.

$$a_{1y} = -a_{2x} = -a$$

$$\begin{cases} T_1 = m_1 g = m_1 a_{1y} \\ T_2 = m_2 a_{2x} \\ T_1 = T_2 \\ a_{1y} = -a_{2x} = -a \end{cases} \Rightarrow$$

$$a = \frac{m_1}{m_1 + m_2} g$$

$$\text{se } m_2 \rightarrow \infty \Rightarrow a = g$$

$$T = m_2 a_{2x} = \frac{m_2 m_1}{m_1 + m_2} g$$

$$a_{1y} = -\frac{m_1}{m_1 + m_2} g$$

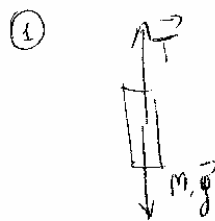
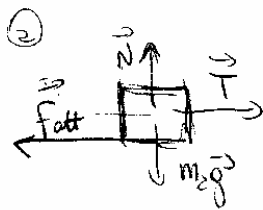
$$a_{2x} = \frac{m_1}{m_1 + m_2} g$$

il moto è uniform. acc.

$$y_1^i(t) = y_1^i(0) + \underbrace{v_{1y}^i(0)}_0 t - \frac{1}{2} \left(\frac{m_1}{m_1 + m_2} g \right) t^2$$

$$x_2(t) = x_2(0) + \underbrace{v_{2x}(0)}_0 t + \frac{1}{2} \left(\frac{m_1}{m_1 + m_2} g \right) t^2$$

$$\Delta x_2^i = -\Delta y_1^i \quad \leftarrow \text{per verifica}$$



$$|\vec{F}_{att}| \leq N \mu_s$$

$$\textcircled{2} \quad \lambda: T - F_{att} = m_2 a_{2x}$$

$$\uparrow: N - m_2 g = 0 \quad \rightarrow N = m_2 g$$

$$F_{att} = T - m_2 a_{2x} \leq \mu_s m_2 g$$

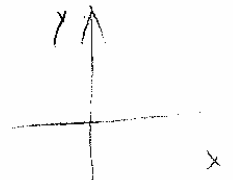
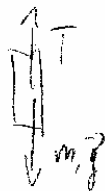
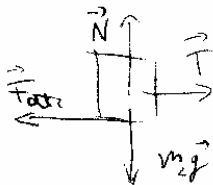
caso limite $a_{2x} = 0$ quindi

$$F_{att} = T = \mu_s^* m_2 g$$

$T - m_2 g = 0$ ← la ricavato dal ② corpo
perché sono nelle condiz. limite

$$\mu_s^* = \frac{m_1}{m_2}$$

3. $\mu_s < \mu_s^* = \frac{m_1}{m_2} \rightarrow \mu_s$ parte e μ_d quindi m_2 inizia a scivolare



① $\hat{y}: T - m_1 g = m_1 a_{1y}$

② $\hat{x}: T - F_{att}^d = m_2 a_{2x}$

$\hat{y}: N_2 - m_2 g = 0 \rightarrow N_2 = m_2 g$

$$F_{att}^d = \mu_d N = \mu_d m_2 g$$

$$-a_{1y} = +a_{2x} = a$$

$$-m_1 g + F_{att}^d = -m_1 a - m_2 a$$

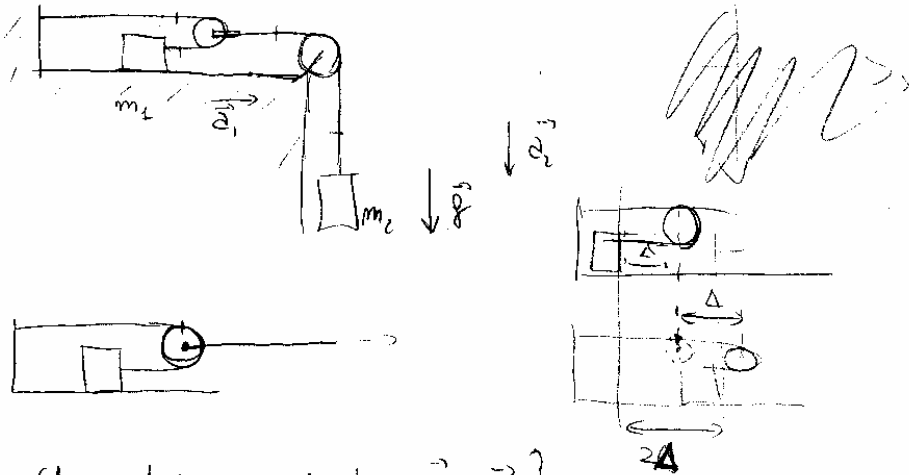
$$-m_1 g + \mu_d m_2 g = -a (m_1 + m_2)$$

$$a = \frac{m_1}{m_1 + m_2} g - \mu_d \frac{m_2}{m_1 + m_2} g$$

vediamo se ha senso: se $\mu_d = 0 \rightarrow$ ho le soluz. di prima
e $\mu_d \neq 0 \rightarrow a < a \text{ di prima}$

ESERCIZIO 3

$$\mu_s = \mu_d = 0$$



- 1) Che relazione c'è tra $\vec{a}_1$ e $\vec{a}_2$?
- 2) Tensioni delle funi nei vari tratti
- 3) Determinare a_1 e a_2 in funzione di m_1 e m_2

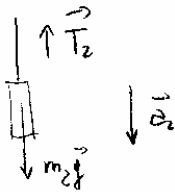
$$\Delta y_2 = \Delta$$

$$\Delta x_1 = 2\Delta$$

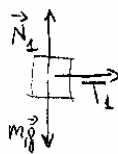
faccio la derivata seconda e trovo che:

$$a_1 = 2a_2$$

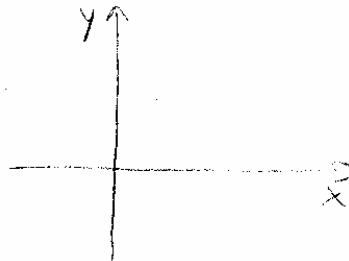
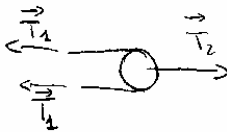
①



③



②



②

$$\hat{x} \cdot T_2 - 2T_1 = 0 \quad \leftarrow \text{perché } m \text{ della carrucola è zero} \quad \rightarrow \quad T_1 = \frac{T_2}{2}$$

①

$$\hat{y}: T_2 - m_2 g = -m_2 a_2$$

③

$$\hat{y} \left\{ \begin{array}{l} N_1 = m_2 g \\ T_1 = m_1 a_1 \end{array} \right.$$

$$2T_1 = m_2 (g - a_2)$$

$$T_1 = m_1 a_1$$

$$2m_1 a_1 = m_2 (g - a_2)$$

$$4m_1 a_2 = m_2 (g - a_2)$$

$$a_1 = 2a_2$$

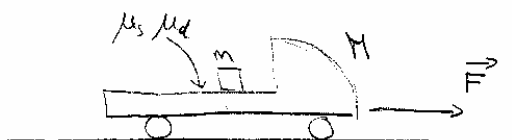
$$a_2 = \frac{m_2}{(4m_1 + m_2)} g$$

$$a_1 = \frac{2m_2}{(4m_1 + m_2)} g$$

$$T_1 = \frac{2m_1 m_2}{(4m_1 + m_2)} g$$

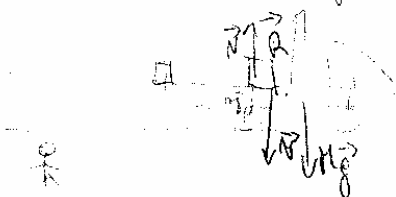
$$T_2 = \frac{m_1 m_2}{(4m_1 + m_2)} g$$

ESERCIZIO 41

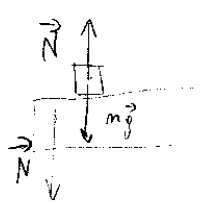


1) $\mu_s = \mu_d = 0$

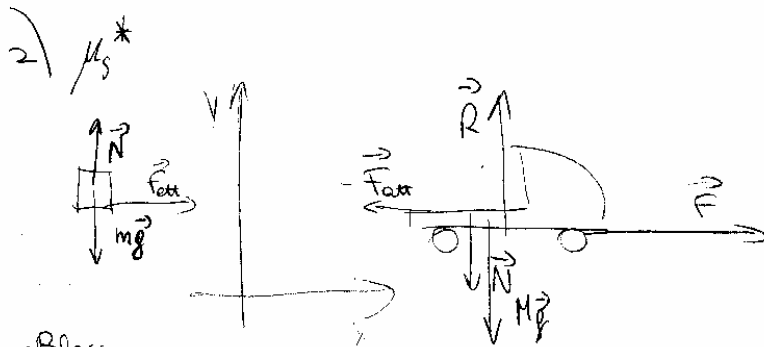
1.1 determinare $\vec{a}_{\text{person}}$ e $\vec{f}_b$ - due forze sul blocco



$\hat{x}: F = M a_f$ sul furgone



$$a = \frac{F}{M}$$



Blocco

$$\begin{cases} \hat{x}: F_{att} = m a_x \\ \hat{y}: N - mg = 0 \end{cases}$$

Furgone

$$\begin{cases} \hat{x}: F - F_{att} = M a_x \\ \hat{y}: R - N - Mg = 0 \end{cases}$$

furgone e blocco non scivolano,
si muovono in maniera solidale,
quindi $a_x = a_x = a$
 a_f a_{blocco}

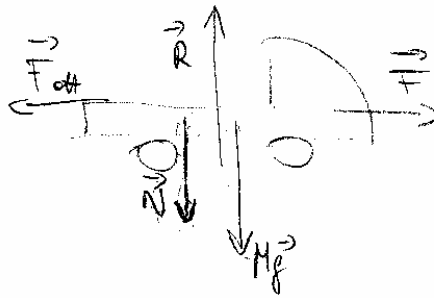
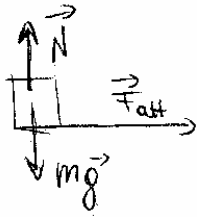
$$F_{att} \leq N \mu_s$$

condiz. limite : $F_{att} = N \mu_s^* = mg \mu_s^*$

$$F = (m + M) a \rightarrow a = \frac{F}{m + M}$$

$$F_{att} = mg \mu_s^* \rightarrow F - mg \mu_s^* = \frac{M F}{m + M} \rightarrow \mu_s^* = \frac{F}{(M + m) g}$$

3) $\mu_s < \mu_s^*$ $\left(a_{\text{blocco}} < a_{\text{appena calcolata}} \right)$



Blocco

$$\begin{cases} \hat{x}: F_{\text{att}} = m a_x \\ \hat{y}: N - mg = 0 \end{cases}$$

Furg

$$\begin{cases} \hat{x}: F - F_{\text{att}} = M A_x \\ \hat{y}: R - N - Mg = 0 \end{cases}$$

c'è scorrimento $\Rightarrow a_x \neq A_x$

$$F_{\text{att}} = \mu_d N = \mu_d mg$$

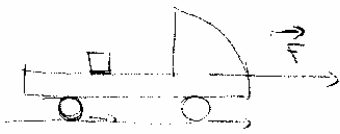
$$a_x = \mu_d g$$

$$F - \mu_d mg = M A_x$$

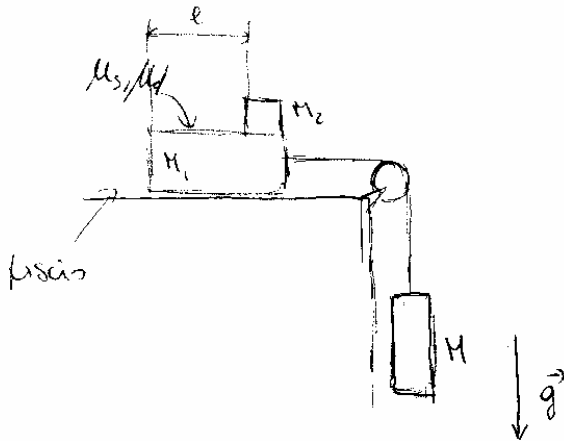
$$A_x = \frac{F}{M} - \mu_d \frac{m}{M} g$$

$$\vec{F}_{\text{att}} = mg \mu_d \hat{x}$$

24/03/06 (3 ore)



il motore gira sul terreno
e scivola che porta al attito minimo

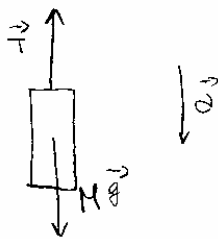


DATI:

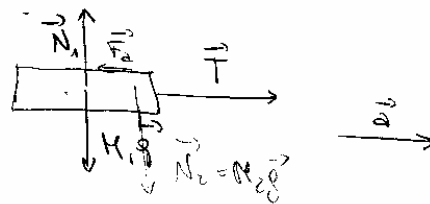
M_1, M_2, μ_s, μ_d

- 1) qual è la condiz su M , affinché non ci sia scorrimento tra M_1 e M_2
- 2) Se la condiz. $\uparrow$ non è soddisfatta, trovare le $\vec{a}$ di M
 $\vec{a}_1$ di M_1
 $\vec{a}_2$ di M_2
- 3) Nelle stesse ipotesi della 2, dopo quanto tempo t^* , M_2 arriva alla fine di M_1 .

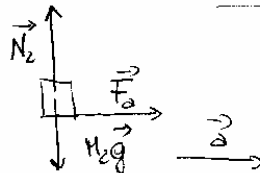
① (M)



② (M2)



③ (M1)



perché siamo nell'ipotesi che non
ci sia scorrimento

Equazioni del moto

$$\begin{aligned}
 \textcircled{M} \quad \hat{y}: T - M g &= -M a \\
 \textcircled{M_1} \quad \hat{x}: T - \vec{F}_a &= M_1 a \\
 \hat{y}: N_1 - \underbrace{N_2}_{M_2 g} - M_1 g &= 0 \rightarrow N_1 = (M_1 + M_2) g \\
 \textcircled{M_2} \quad \hat{x}: F_a &= M_2 a \rightarrow a = \frac{F_a}{M_2} \\
 \hat{y}: N_2 - M_2 g &= 0 \rightarrow N_2 = M_2 g
 \end{aligned}$$

$$N_1 = (M_1 + M_2) g$$

$$M g - F_a = (M + M_1) \frac{F_a}{M_2}$$

$$\Rightarrow F_a = \frac{M M_2}{M + M_1 + M_2} g \leq \mu_s N_2 = \mu_s M_2 g$$

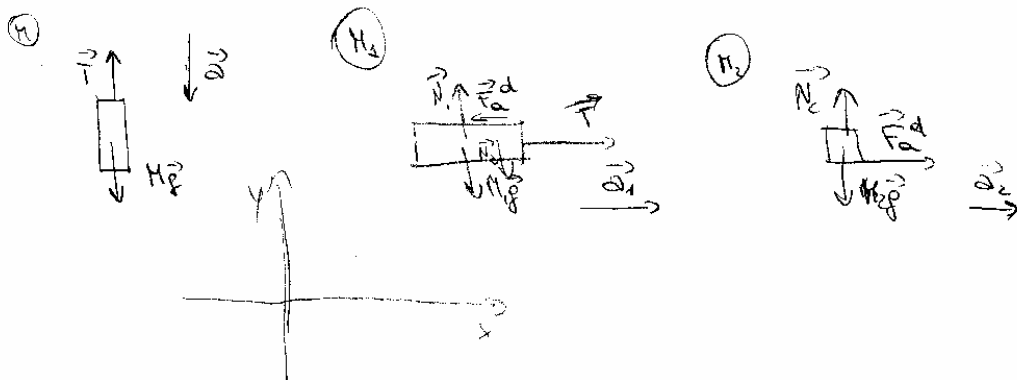
$$\text{caso limite } F_a = \mu_s M_2 g = \frac{M M_2}{M + M_1 + M_2} g$$

$$\Rightarrow M = \mu_s (M + M_1 + M_2) \rightarrow \boxed{M = \frac{\mu_s (M_1 + M_2)}{1 - \mu_s}}$$

se $M \leq \frac{\mu_s (M_1 + M_2)}{1 - \mu_s}$
l'attrito è statico,
se $M > \frac{\mu_s (M_1 + M_2)}{1 - \mu_s}$
l'attrito è dinamico

$$M \leq \frac{\mu_s}{1 - \mu_s} (M_1 + M_2)$$

$$\textcircled{2} \text{ If } M > \frac{\mu_s}{1 - \mu_s} (M_1 + M_2)$$



$$\textcircled{M} \quad \hat{y}: T - Mg = -Ma$$

$$\textcircled{H_1} \quad \hat{x}: T - F_a^d = M_1 a_1$$

$$\hat{y}: N_1 - N_2 - M_1 g = 0$$

$a_1 = 0$ perché il filo

$F_a^d = \mu_d N_1 =$ è invertito

$$= \mu_d N_2 =$$

$$= \mu_d M_2 g$$

$$\textcircled{H_2} \quad \hat{x}: F_a = M_2 a_2$$

$$\hat{y}: N_2 - M_2 g = 0 \longrightarrow N_2 = M_2 g$$

$$a_2 = \mu_d g$$

$$Mg - F_a^d = (M + M_1) a$$

$$Mg - \mu_d M_2 g = (M + M_1) a \rightarrow a = \frac{(M - \mu_d M_2)}{M + M_1} g$$

- devo verificare che $a > 0 \Rightarrow M > \mu_d M_2$

seppur anche
 $\mu_s > \mu_d$

$$M > \frac{\mu_s}{1 - \mu_s} (M_1 + M_2) > \mu_d M_2$$

Devo

$$M > \mu_s (M + M_1 + M_2) > \mu_d M_2$$

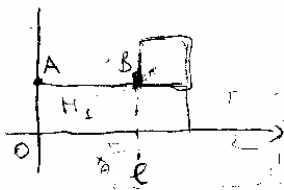
- verificare che $a > a_2$ ~~abbastanza~~ "buon senso"

$$\frac{M - \mu_d M_2}{M + M_1} > \mu_d$$

$$M > \mu_d (M + M_1 + M_2) \quad \text{è vero perché so che:}$$

$$M > \mu_s (M + M_1 + M_2) \quad \text{ed} \quad \mu_s > \mu_d$$

3)



$$x_A(t) = \frac{1}{2} a t^2$$

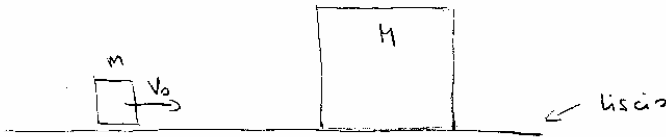
$$x_B(0) = l$$

$$x_B(t) = l + \frac{1}{2} a_2 t^2$$

$$x_A(t^*) = x_B(t^*)$$

$$\frac{1}{2} a t^2 = l + \frac{1}{2} a_c t^2$$

$$\Rightarrow t^* = \sqrt{\frac{2l}{a-a_c}} \quad a > a_c$$



trovare le vel.
un istante dopo l'urto

1) Urto elastico

2) Urto anelastico

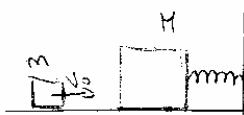
$$1) \begin{cases} m v_0 = m v + M V \\ \frac{1}{2} m v_0^2 = \frac{1}{2} m v^2 + \frac{1}{2} M V^2 \end{cases}$$

$$\left(\frac{M}{m} + 1 \right) V^2 = 2 v_0^2$$

$$V = 0 \rightarrow$$

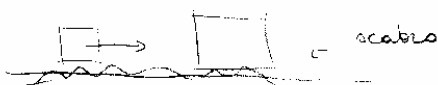
$$\begin{cases} V = \frac{2 v_0 m}{(M+m)} \\ v = \frac{(m-M)}{m+M} v_0 \end{cases}$$

se $m = M \Rightarrow m$ si ferma e M parte
con $v = v_0$



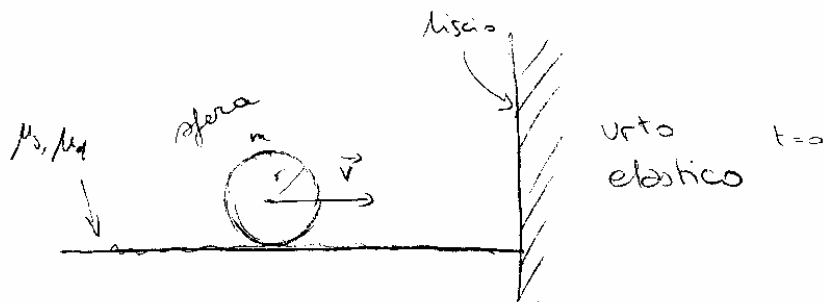
$\vec{P}$ si conserva

la Fe non dà impulso
significativo



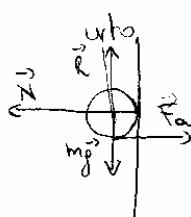
$\vec{P}$ si conserva

la F attrito non è
impulsiva



Determinare la legge oraria per $t > 0$
rotolamento puro $\rightarrow$ cioè μ_s

$v = \omega r$



N è impulsivo

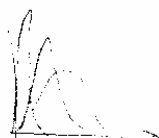
$\hat{y}: R - mg = 0 \rightarrow R = mg$ sicuramente non è impulsiva

$\hat{x}: F - N = m a$

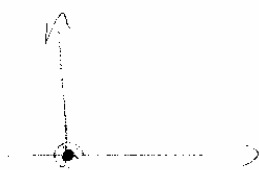
$F \approx \mu N = \mu mg$
 R

F non è impulsiva (ha un valore finito)

N è impulsiva



$\vec{v} = v \hat{x}$
 $\vec{\omega} = -\omega \hat{z}$



$\hat{z}$ è scelto per come abbiamo scelto gli assi

$E_i = \frac{1}{2} m v^2 + \frac{1}{2} I_{cm} \omega^2$

$\omega = \frac{v}{R}$

$E_f = \frac{1}{2} m v'^2 + \frac{1}{2} I_{cm} \omega'^2$

N passa per cm , quindi ha $M = 0$ $\Rightarrow L$ si conserva

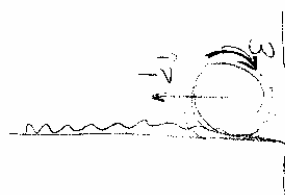
$\Rightarrow E_i = E_f \Rightarrow v^2 = v'^2$

$\omega = \omega'$

$\Rightarrow \omega$ si conserva

$v' = \pm v$

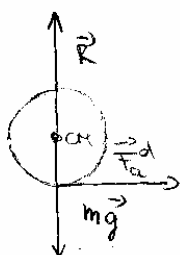
se $V' = V \rightarrow$ la sfera attraversa il muro
 $V' = -V$



ω è rimasta uguale anche nel verso

ora l'attrito è dinamico

se la parete era scabra, l'attrito sulla parete era una forza impulsiva che aveva un momento



Equib. moto

I^e equaz. cardinale

$$\hat{x}: F_d = m a_x$$

$$F_d = \mu_d N = \mu_d R = \mu_d mg$$

$$\hat{y}: R - mg = 0$$

II^e equaz. cardinale

$$\vec{\omega} = -\omega \hat{z}$$

$$\sum_{i=1}^n \vec{r}_i \wedge \vec{F}_i = I \frac{d\vec{\omega}}{dt}$$

$$r F_d = I_{cm} \frac{d\omega}{dt}$$

$$F_d = \mu_d mg \quad F_d = \mu_d mg$$

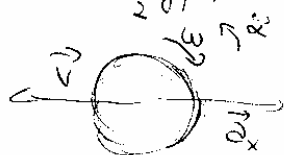
$$a_x = \mu_d g$$

$$r \mu_d mg = I_{cm} \frac{d\omega}{dt}$$

$$I_{sfera} = \frac{2}{5} MR^2$$

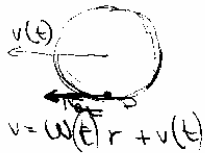
$$\dot{\alpha} = \frac{5}{2} g \mu_d$$

$$\omega(t) = \int_0^t \alpha(t') dt' + \omega(0)$$



$$\omega(t) = \int_0^t \alpha(t') dt' + \omega(0) = \frac{5g}{2r} \mu_d t - \frac{v}{r}$$

$$v(t) = \alpha_x t + v(0) = \mu_d g t - v$$

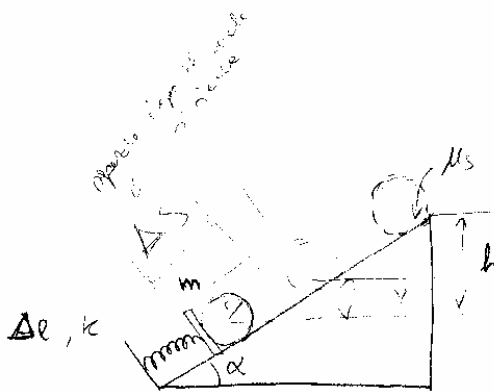


$$v_p = 2v - \frac{7}{2} \mu_d g t$$

$$t = t^* \quad v_p = 0$$

$$t^* = \frac{4v}{7\mu_d g}$$

la sfera smette
di strisciare e
inizia moto
rotatorio puro



h

$$1) \mu_s = 0$$

$$2) \mu_s \neq 0 \text{ suff. da avere rotolamento puro}$$

$$3) \mu_s^* \text{ min affinché ci sia rotolamento puro}$$

$$④ h = ? \quad \text{il cilindro non rotola, trasla}$$

$$E_i = \cancel{\phi} + \cancel{\phi} + \frac{1}{2} k (\Delta l)^2$$

$$E_f = mgh + \cancel{\phi} + \cancel{\phi}$$

$$h = \frac{1}{2} \frac{k(\Delta l)^2}{m g}$$

$E_i = \frac{1}{2} k(\Delta l)^2$ $E_f = \frac{1}{2} m v_0^2 + m g \Delta l \sin \alpha$ $v_0 = \sqrt{\dots}$	$\mu_s \neq 0$ $E_f = \frac{1}{2} m v_0^2 + \frac{1}{2} I \omega_0^2 + m g \Delta l \sin \alpha$ $v_0 = \omega_0 r$ $v_0 = \dots$
---	--

2) $E_i = E_f$ posso ancora usare perché l'attrito è statico e non compie lavoro

$$E_i = \frac{1}{2} k(\Delta l)^2 \Rightarrow h = \frac{1}{2} \frac{k(\Delta l)^2}{m g}$$

$$E_f = m g h$$

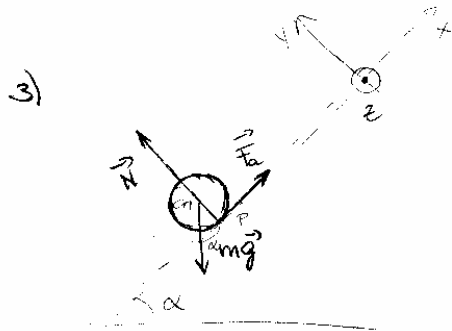
in un punto intermedio

$$E_y = m g y + \frac{1}{2} m v_y^2 + \frac{1}{2} I_{cm} \omega^2 = E_i$$

$\downarrow$
 $\frac{v_y}{r}$

$$v_y = \sqrt{\frac{2}{3} \frac{k}{m} (\Delta l)^2 - \frac{4}{3} g y}$$

$$I_{cm} = \frac{1}{2} M R^2$$



I equaz. cardinali

$$\hat{x}: F - m g \sin \alpha = m a_x$$

$$\hat{y}: N - m g \cos \alpha = 0$$

II equaz. cardinale $\rightarrow$ rispetto all'asse baricentrico o al polo fisso di contatto

$$\textcircled{cm} \quad r F_a = I_{cm} \frac{d\omega}{dt}$$

rispetto al polo mobile dell'asse baricentrico
(l'unica forza che produce momento è F_a)

$$\textcircled{P} \quad r m g \sin \alpha = (I_{cm} + m r^2) \frac{d\omega}{dt}$$

rispetto al polo P

(l'unica forza che produce momento è $m g \sin \alpha$)

Scegliamo CM

accelerazione angolare
 $\alpha r = a_x$

$$F = mg \sin \alpha + m a_x = mg \sin \alpha - m \frac{d\omega}{dt} r$$

altimenti ω aumentano
in modulo, ma nel verso
negativo

partitisco in

CM

$$r m g \sin \alpha - r^2 m \frac{d\omega}{dt} = I_{CM} \frac{d\omega}{dt}$$

da P

$$\frac{d\omega}{dt} = \frac{r m g \sin \alpha}{I_{CM} + r^2 m} = \frac{r m g \sin \alpha}{\frac{3}{2} m r^2} = \frac{2 g \sin \alpha}{3 r} \quad \text{accelerazione angolare}$$

$$a_x = -r \frac{d\omega}{dt} = -\frac{2}{3} g \sin \alpha$$

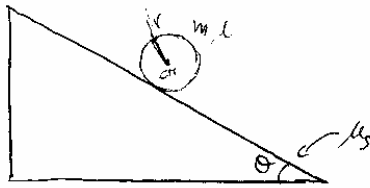
$$\Rightarrow F_a^0 = mg \sin \alpha - \frac{2}{3} mg \sin \alpha = \frac{1}{3} mg \sin \alpha \leq \mu_s \cdot N = \mu_s mg \cos \alpha$$

$$\mu_s^* = \frac{1}{3} \tan \alpha$$

$$\frac{1}{3} mg \sin \alpha = \mu_s^* mg \cos \alpha$$

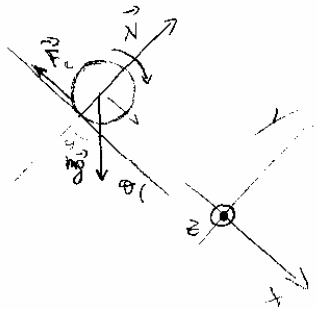
24/03/06 (3.12) esercizi

ESERCIZIO 1



rotolamento puro

Calcolare l'accelerazione del C.M., a



I equaz. coordinate

$$\hat{x}: mg \sin \theta - F_f = m a_x$$

$$\hat{y}: N - mg \cos \theta = 0$$

II equat. rispetto all'asse baricentrico

$$\hat{z}: M_z = \frac{d}{dt} L_z$$

l'unica forza che produce momento è F_f

$$-r F_f = I_{cm} \frac{d\omega}{dt}$$

perché è anticlockwise

$v_x = -r\omega$, perché, per avere una v_x positiva il corpo deve ruotare in senso orario, quindi ω deve essere negativo

$$a_x = -r \frac{d\omega}{dt}$$

$$-r F_f = -\frac{I_{cm}}{r} a_x \quad \rightarrow \quad a_x = \frac{r^2 F_f}{I_{cm}}$$

$$mg \sin \theta - F_f = \frac{r^2 F_f}{I_{cm}}$$

$$F_f \left(\frac{r^2}{I_{cm}} + \frac{1}{m} \right) = g \sin \theta \quad \rightarrow \quad F_f = \frac{I_{cm} m g \sin \theta}{I_{cm} + m r^2}$$

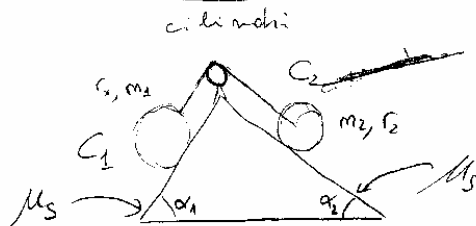
$$a_x = g \sin \theta \left(1 - \frac{I_{cm}}{I_{cm} + mr^2} \right) =$$

$$I_{cm} = \frac{1}{2} mr^2$$

$$a_x = \frac{2}{3} g \sin \theta$$

non si poteva usare la conservazione dell'energia perché non compare mai l'accelerazione.

ESERCIZIO 2



~~Accelerazione~~

rotolamento puro

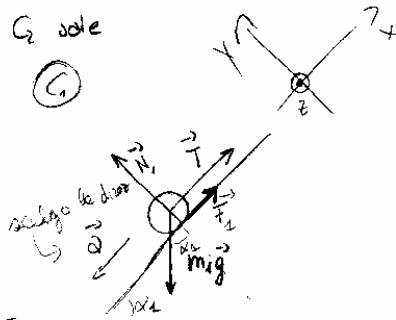
Accelerazione del 1° cilindro (C)

ma ci sono accelerazioni da trovare che sono le forze

C1 scende

C2 sale

(C)



I coord

$$\hat{x}: T + F_1 - m_1 g \sin \alpha_1 = -m_1 a_1$$

$$\hat{y}: N_1 - m_1 g \cos \alpha_1 = 0$$

II coord

$$\hat{z}: r_1 F_1 = I_{cm1} \frac{d\omega_1}{dt}$$

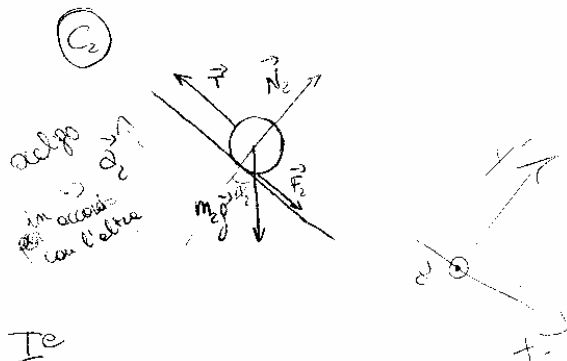
$$a_1 = r_1 \frac{d\omega_1}{dt}$$

$$r_1 F_1 = I_{cm1} \frac{a_1}{r_1}$$

$$F_1 = I_{cm1} \frac{a_1}{r_1^2}$$

$$T - m_1 g \sin \alpha_1 + \frac{I_{cm1} a_1}{r_1^2} = -m_1 a_1$$

$$\left(\frac{I_{CM1}}{r^2} + m_1 \right) a_1 = m_1 g \sin \alpha_1 + T$$



$$a_1 = a_2 = a$$

Ic

$$\hat{x}' : m_2 g \sin \alpha_2 - T + F_2 = -m_2 a_2$$

$$\hat{y}' : N_2 - m_2 g \cos \alpha_2 = 0$$

IIa

$$\hat{z} : r^2 F_2 = I_{CM2} \frac{d\omega_2}{dt}$$

$$a_2 = r_2 \frac{d\omega_2}{dt}$$

$$r_2^2 F_2 = I_{CM2} \frac{a_2}{r_2} \rightarrow F_2 = \frac{I_{CM2} a_2}{r_2^2}$$

$$\left(\frac{I_{CM2}}{r^2} + m_2 \right) a_2 = T - m_2 g \sin \alpha_2$$

$$a \left(\frac{I_{CM1}}{r^2} + m_1 + \frac{I_{CM2}}{r^2} + m_2 \right) = g (m_1 \sin \alpha_1 - m_2 \sin \alpha_2)$$

$$I_{CM} = \frac{1}{2} m r^2$$

$$a \left(\frac{m_1}{2} + m_1 + \frac{m_2}{2} + m_2 \right) = g (m_1 \sin \alpha_1 - m_2 \sin \alpha_2)$$

$$a = \frac{g (m_1 \sin \alpha_1 - m_2 \sin \alpha_2)}{\frac{3}{2} (m_1 + m_2)}$$

~~Di punto~~

(i) fermi

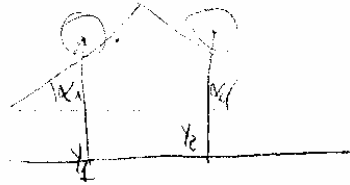
(f) C_1 è verso di ΔS

Di quanto cambia la velocità del C_{H_1} , C_{H_2} , ω_1 , ω_2 e cc.

$$E_i^{\text{tot}} = E_f^{\text{tot}}$$

$$E_i^{\text{tot}} = m_1 g y_1 + m_2 g y_2$$

~~tot~~



$$E_f^{\text{tot}} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} I_{\text{cm}1} \omega_1^2 + m_1 g (y_1 - \Delta S \sin \alpha_1) + \\ + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} I_{\text{cm}2} \omega_2^2 + m_2 g (y_2 + \Delta S \sin \alpha_2)$$

$$v_1 = \omega_1 r_1 = v_2 = \omega_2 r_2 = v$$

~~tot~~

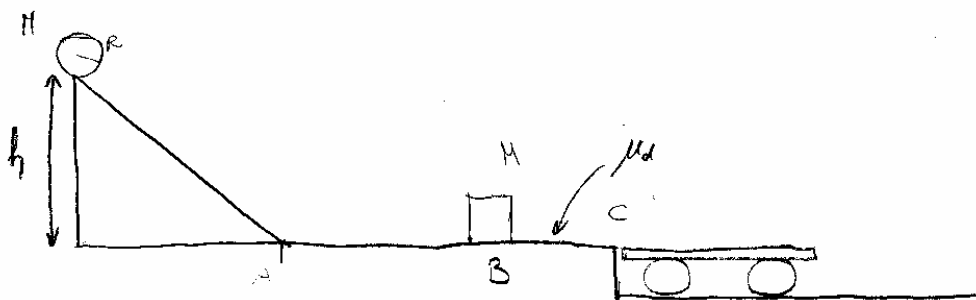
$$\frac{1}{2} \left(m_1 + \frac{I_{\text{cm}1}}{r_1^2} \right) v^2 + m_2 g \Delta S \sin \alpha_2 - m_1 g \Delta S \sin \alpha_1 = 0$$

$$I_{\text{cm}} = \frac{1}{2} m r^2$$

$$v = \sqrt{\frac{4}{3} \Delta S g \frac{(m_1 \sin \alpha_1 - m_2 \sin \alpha_2)}{m_1 + m_2}}$$

ESERCIZIO

andare su internet sul web del Leporini



1.1.

$$E_i = M g (h + R)$$

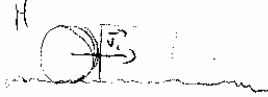
$$E_f = M g R + \frac{1}{2} M v_1^2 + \frac{1}{2} I_{\text{cm}} \omega_1^2$$

$$\omega_1 = \frac{v_1}{R}$$

$$I_{\text{cm}} = \frac{2}{5} M R^2$$

$$v_1 = \sqrt{\frac{10}{7} g h}$$

1.2



usiamo la conservazione delle quantità di moto, anche se c'è attrito, ~~che~~ non è una forza impetuosa

$$M v_1 = (M + M) v_2$$

$$v_2 = \frac{v_1}{2}$$

$$x_C = L$$

1.3

$v_3 \rightarrow$ uso il teorema delle forze vive

$$W_{BC} = E_{cinetica C} - E_{cinetica B} = \frac{1}{2} (M + M) v_3^2 - \frac{1}{2} (M + M) v_2^2$$



$$\Rightarrow v_3 = \sqrt{v_2^2 - 2 M g L}$$

$$L = \overline{BC}$$

vale fino quando il blocco si ferma

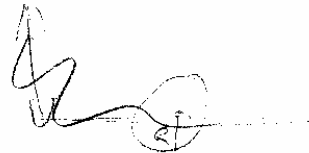
1.5

$$v_4 = \frac{2}{3} v_3$$

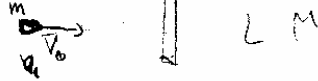
$$E_{dissipata} = - (E_f^{TOT} - E_i^{TOT}) = - \frac{1}{2}$$

$$= - \left[\frac{1}{2} (3M) v_4^2 + \cancel{M g h} - M g h \right]$$

si vede

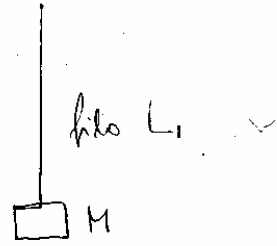


① fulcro



urto perfetto anelastico

②



si conserva le quantità di moto

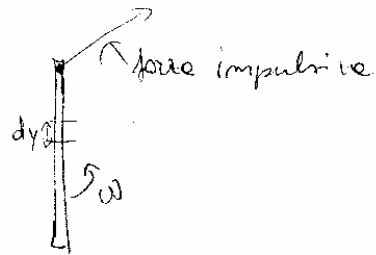
$\omega = ?$ si conserva il
 $v = ?$ momento delle quantità di moto
di moto
ma non le quantità di moto

①

$$L_{0z} = L_z$$

$$L_{0z} = L m v_0$$

dopo l'urto



non è possibile proiettare se non c'è la sbarra

$$L_z = I_z \omega + L m v$$

$$I_z = \int dm r^2$$

$r = y$

rispetto all'asse del fulcro

$$\frac{M}{L} = \frac{dm}{dy} \quad \rightarrow \quad I_z = \int_0^L \frac{M}{L} dy y^2 = \frac{M}{L} \left| \frac{1}{3} y^3 \right|_0^L = \frac{M}{3} L^3$$

$$v = \omega L \quad \rightarrow \quad \omega = \frac{v}{L}$$

$$L m v_0 = \frac{1}{3} M L^2 \frac{v}{L} + L m v$$

$$\rightarrow v = \frac{v_0}{1 + \frac{M}{3m}}$$

② il filo non può trasmettere forze a lui trasversali, quindi il fulcro non può trasmettere forze orizz.

$$m v_0 = (M + m) v$$

$$v = v_0 \frac{m}{M + m}$$

$$v = \frac{v_0}{1 + \frac{M}{m}}$$